Single-degree-of-freedom Plastic Analysis (SPAn32)

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SPAn32

- **SPAn32 was written originally for design and analysis of structural members subjected to high explosive loadings. It performs an equivalent single degree of freedom dynamic analysis of the response of a structural member. It is a useful tool in analysis of conventional construction subjected to any uniform dynamic load.**
- **Web** **https://pdmcx.pecp1.nwo.usace.army.mil/software/span32/**
- **Project Page** **https://forge1.pecp1.nwo.usace.army.mil/projects/span32**

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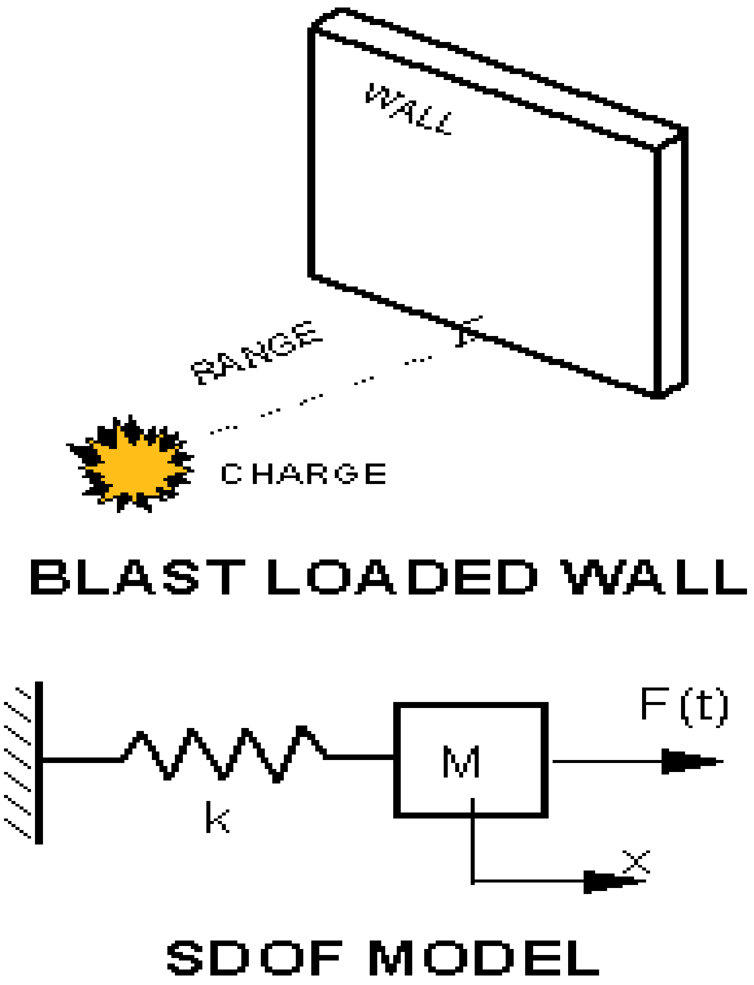
Typical Application of SPAn32

- **Structural wall member is subject to explosive event.**
- **Structural Wall member is modeled as an equivalent Single Degree of Freedom SDOF model.**



Structural Modeling

- **One Way Members**
  - **Steel Beam Model**
  - **Steel Plate Model**
  - **Reinforced Concrete**
- **Two Way Members**
  - **Steel Model**
  - **Reinforced Concrete**
- **User Defined**

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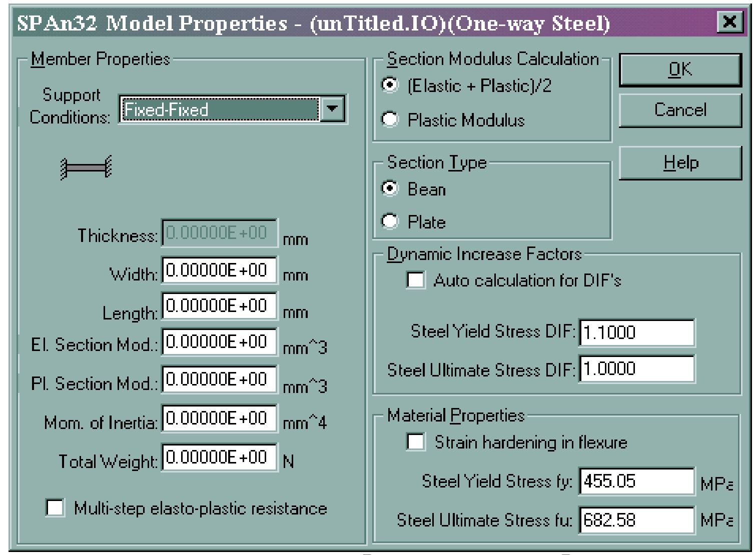
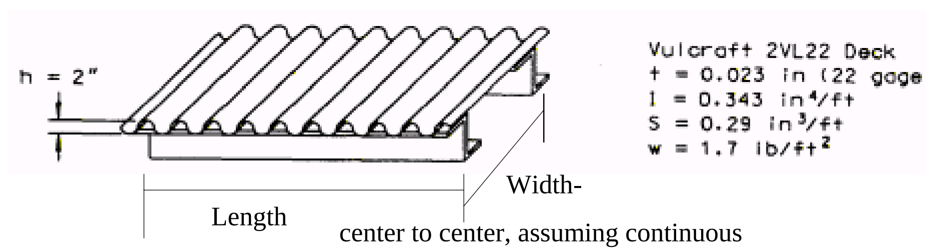
One-Way Steel Beam

- **This is for input of typical steel shapes, AISC or other standard section.**
- **Inputs**
  - **Width – of load acting on member**
  - **Length –**
  - **Elastic Section Modulus**
  - **Plastic Section Modulus**
  - **Moment of Inertia**
  - **Total weight of steel section member whose response is being calculated.**



Purlin Example

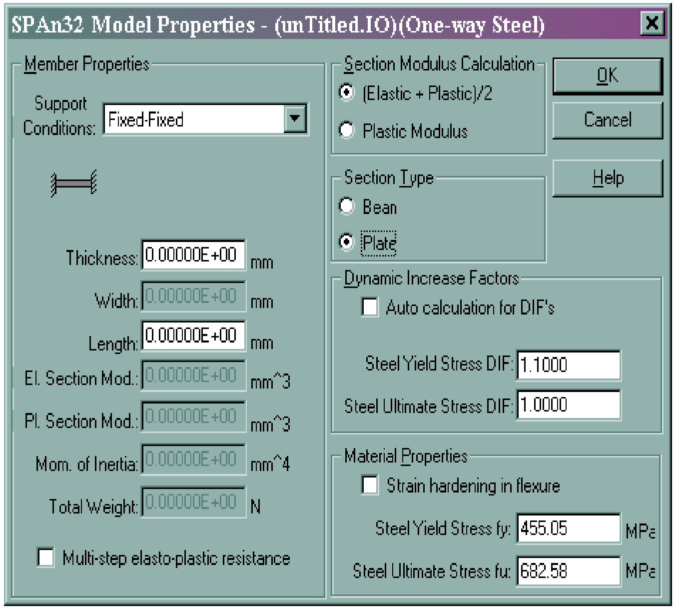


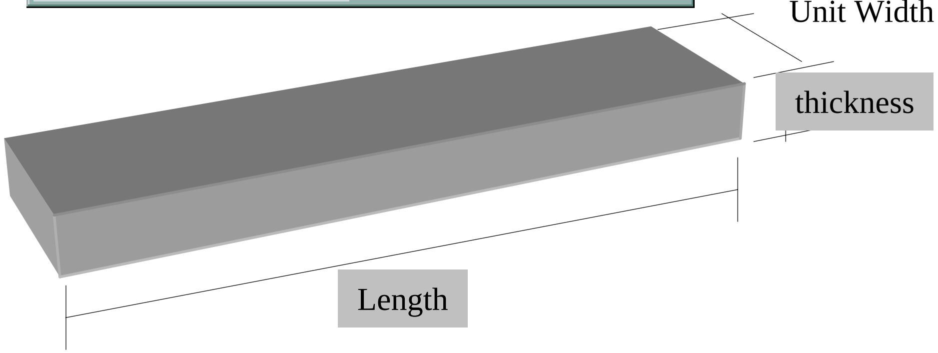
Use Sx, Zx, I, Total Weight of purlin

contribution of decking handled in load definition

One-Way Steel Plate Model

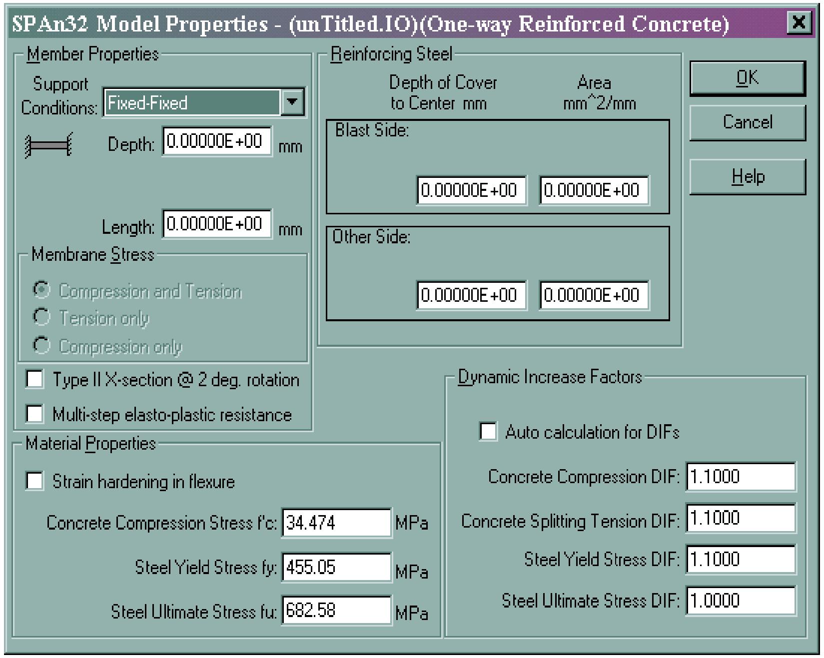
- This is for input of rectangular steel sections of unit width.
- Inputs
  - Thickness of member
  - Length of member
- Load of this member is based on a unit width. In this case it is 1 mm width.
- Resistance – bilinear, Elastic perfectly plastic.

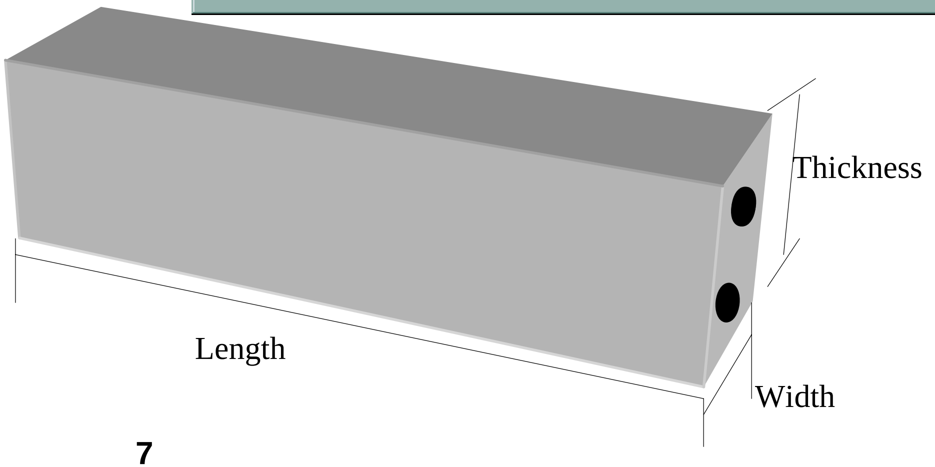




One-Way Reinforced Concrete

- **This for input of reinforced concrete of unit width.**
- **Inputs**
  - **Depth of member**
  - **Length of member**
- **Resistance Models**
  - **Bilinear**
  - **Multi-step**
  - **Type II X-section**
  - **Membrane (T and/or C)**





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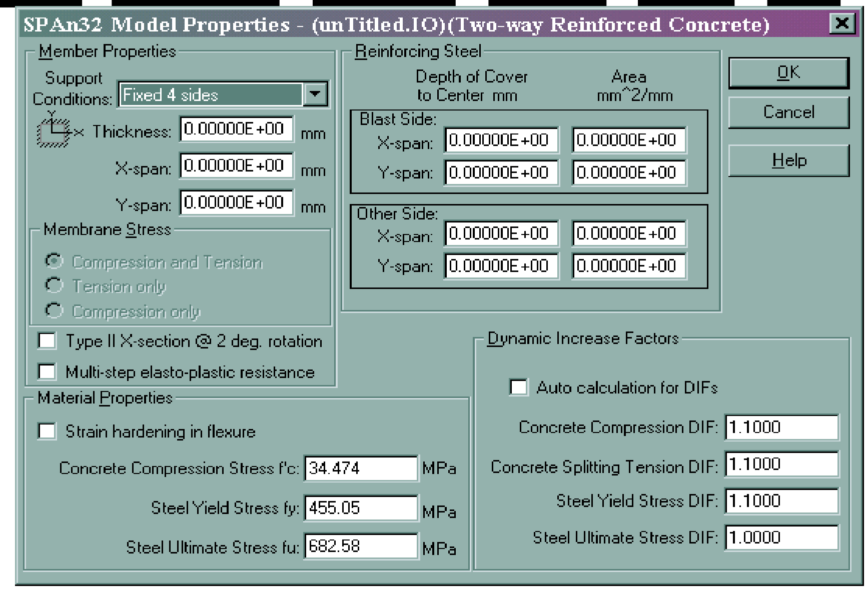
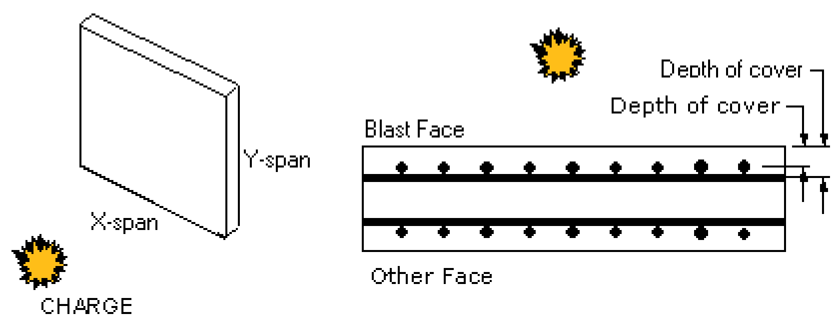
Two-Way Reinforce Concrete Members

- **Reinforced concrete slab/or wall. With reinforcement each way/ each face.**
- **Input**
  - **Thickness**
  - **X-span length**
  - **Y-span length**
- **Resistance**
  - **Bilinear**
  - **Multi-step**
  - **Type II X-section**
  - **Membrane (T and/or C)**





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Model Type Special Features

Model Type Options	Concrete		Steel			User Defined
	1-way	2-way	1-way Beam	1-way Plate	2-way	
Special Features						
Strain Hardening	X	X	X	X	X	
Automatic DIF calculation	X	X	X	X	X	
Deep Beam	X	X				
Concrete Tension DIF	X	X				
Membrane Resistance Tension &/or Compression	X	X				

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Model Type Material Properties

Model Type Options	Concrete		Steel			User Defined
	1-way	2-way	1-way Beam	1-way Plate	2-way	
Material Properties						
Concrete Compressive Str. $f`_c$	X	X				
Steel Yield Str. f_y	X	X	X	X	X	
Steel Ultimate Str. f_u	X	X	X	X	X	
Concrete Splitting Tension DIF	X	X				
Concrete Compressive DIF	X	X				
Steel Yield Strength DIF	X	X	X	X	X	
Steel Ultimate Str. DIF	X	X	X	X	X	
Type II cross section (conc. only)	X	X				

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Model Type Support Conditions

Model Type Options	Concrete		Steel			User Defined
	1-way	2-way	1-way Beam	1-way Plate	2-way	
1 Way Support Conditions						
Fixed-fixed	X		X	X		
Simple-simple	X		X	X		
Fixed-simple	X		X	X		
Cantilever	X		X	X		
Fixed-Fixed Membrane	X					
2 Way Support Conditions						
Fixed 4 sides		X			X	
Fixed 3 sides 1 free		X			X	
Fixed 2 adjacent sides 2 free		X			X	
Fixed 4 sides Membrane		X				
Simple 4 sides					X	
Simple 3 sides 1 free					X	

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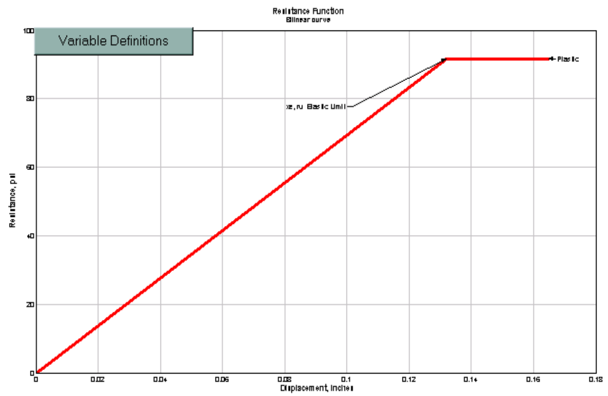
SPAn32 Resistance Function Types

- The resistance function is directly related to the strength and flexibility or stiffness of the structural member.
- The resistance function is composed of elastic and plastic regions, elastic where the member experiences no permanent set, plastic where permanent set occurs.
- Concrete members can model for Compression and Tension membrane effects.

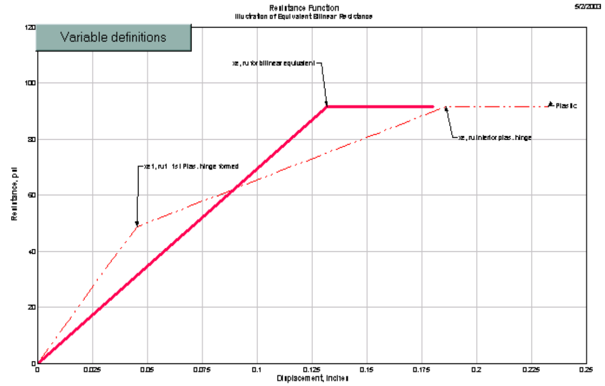
Resistance Functions

- **One-Way and Two-Way reinforced concrete members share the same available resistance function types.**

Bilinear Resistance



Equivalent Bilinear Resistance



Rebound Hysteresis Rules

- **Rebound will always follow the elastic slope.**
- **If stiffness slope is negative (softening stiffness) any rebound will cap the resistance at the resistance where rebound occurs. Rebound is elastic.**
- **If stiffness slope is positive rebound will follow the elastic slope**
- **Positive and negative phase resistance is handled independently. If rebound occurs from the positive phase resistance, the negative phase resistance will load normally accounting for the permanent offset that occurred in the positive phase.**

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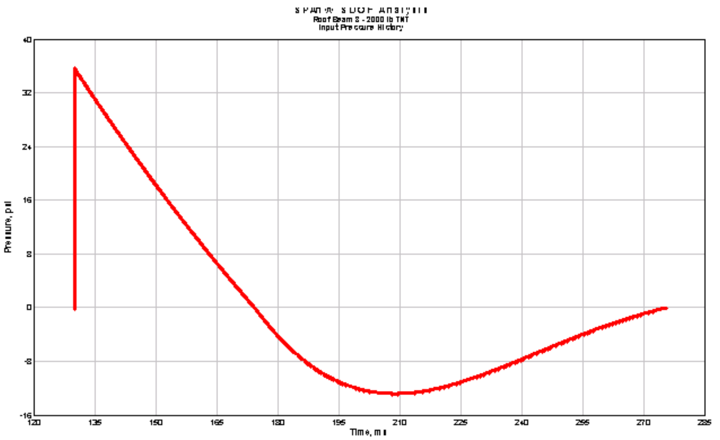
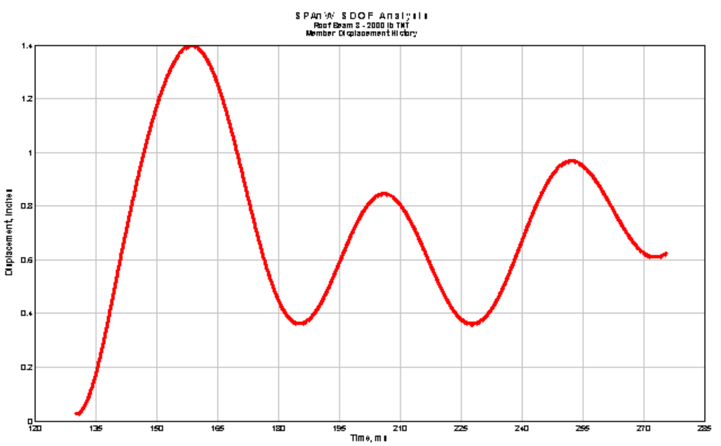
Resistance Functions and Hysteresis Effects

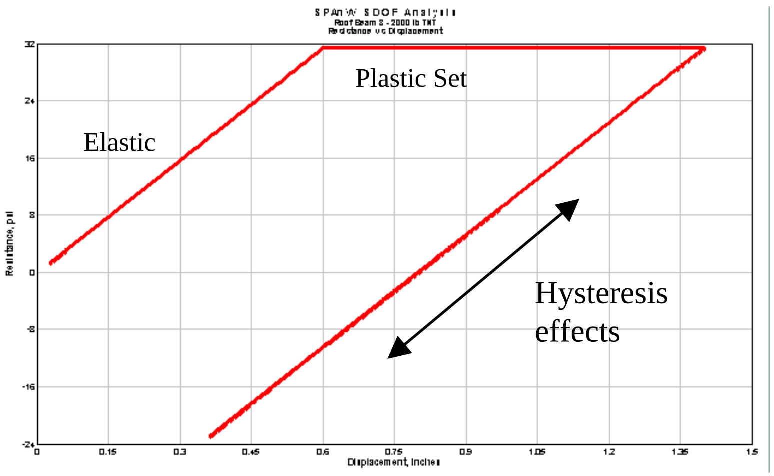
Air Blast Load



Displacement History



Resistance Displacement



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Special Features

- **Deep Member**
- **Membrane Resistance**
- **Dynamic Increase Factor**
- **Strain Hardening**

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Deep Members

- **Deep Member are defined by their support conditions and span to depth ratio.**
  - **L/d < 2.5 Continuous Support**
  - **L/d < 2.0 Simply Supported**
  - **L/d < 5.0 Laterally restrained (Membrane effects)**
- **Deep members behave differently than normally proportioned members, lacking ductility, with special considerations for shear, and flexural response in accordance with UFC 3-340-1 Section 10.5.**

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Reinforced Concrete Membrane Resistance Functions

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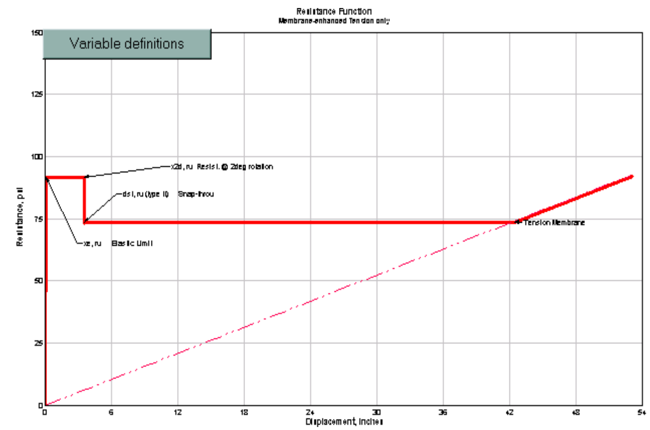
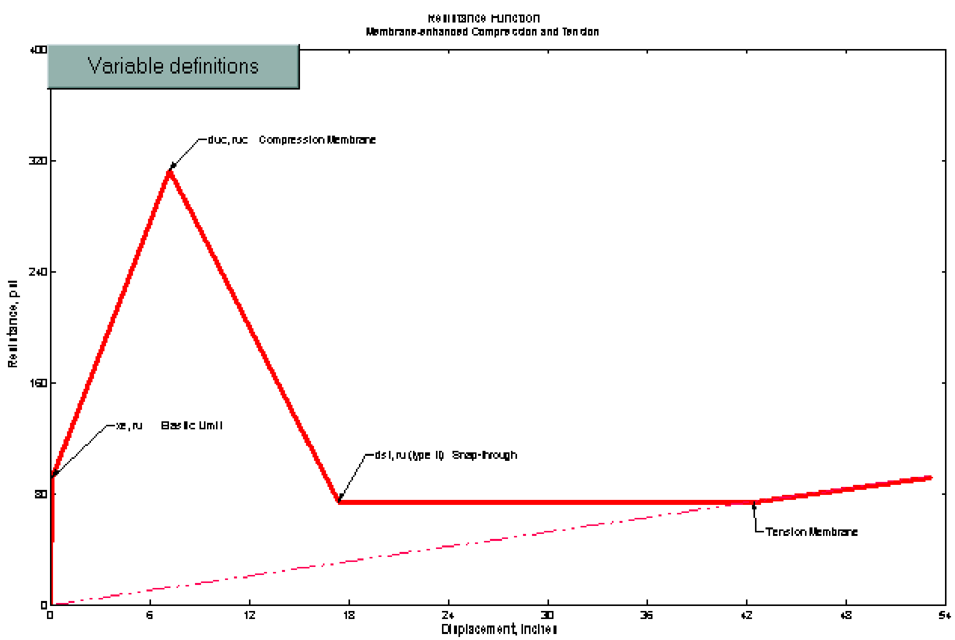
- **One-Way and Two-Way reinforced concrete members share the same available resistance function types.**

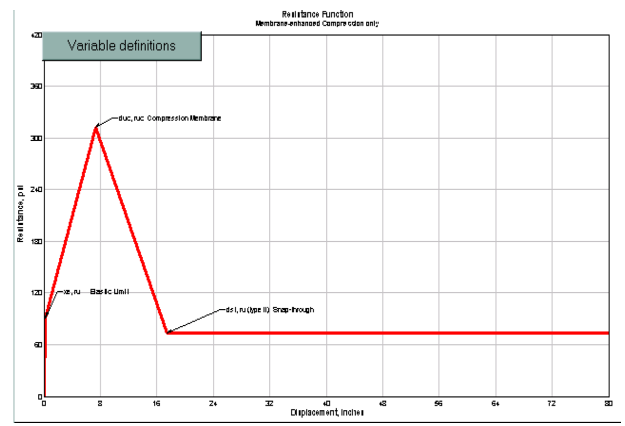
Tension Only



Compression and Tension



Compression Only



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Dynamic Increase Factors

- Both concrete and steel material strengths are strain rate dependent.

- Auto Calculation of Dynamic Increase Factor - A powerful feature in SPAn32 is auto calculation of the dynamic increase factors. SPAn32 estimates the average strain rate to yield in determining dynamic increase factors.

- The material strength dynamic increase factors calculated by SPAn32 are based on the strain rate effect curves in UFC-3-340-1
  - Concrete compression DIF
  - Concrete Splitting Tension DIF
  - Steel Yield Stress DIF
  - DIF Steel Ultimate Stress DIF

Dynamic Increase Factors

- **Concrete Splitting Tension DIF - The effect of this factor on the member is to reduce the need for stirrup shear reinforcing.**

- **Concrete Compression, Steel, and Ultimate Steel Dynamic Increase factors result in larger ultimate resistance value in the resistance function.**

Strain Hardening

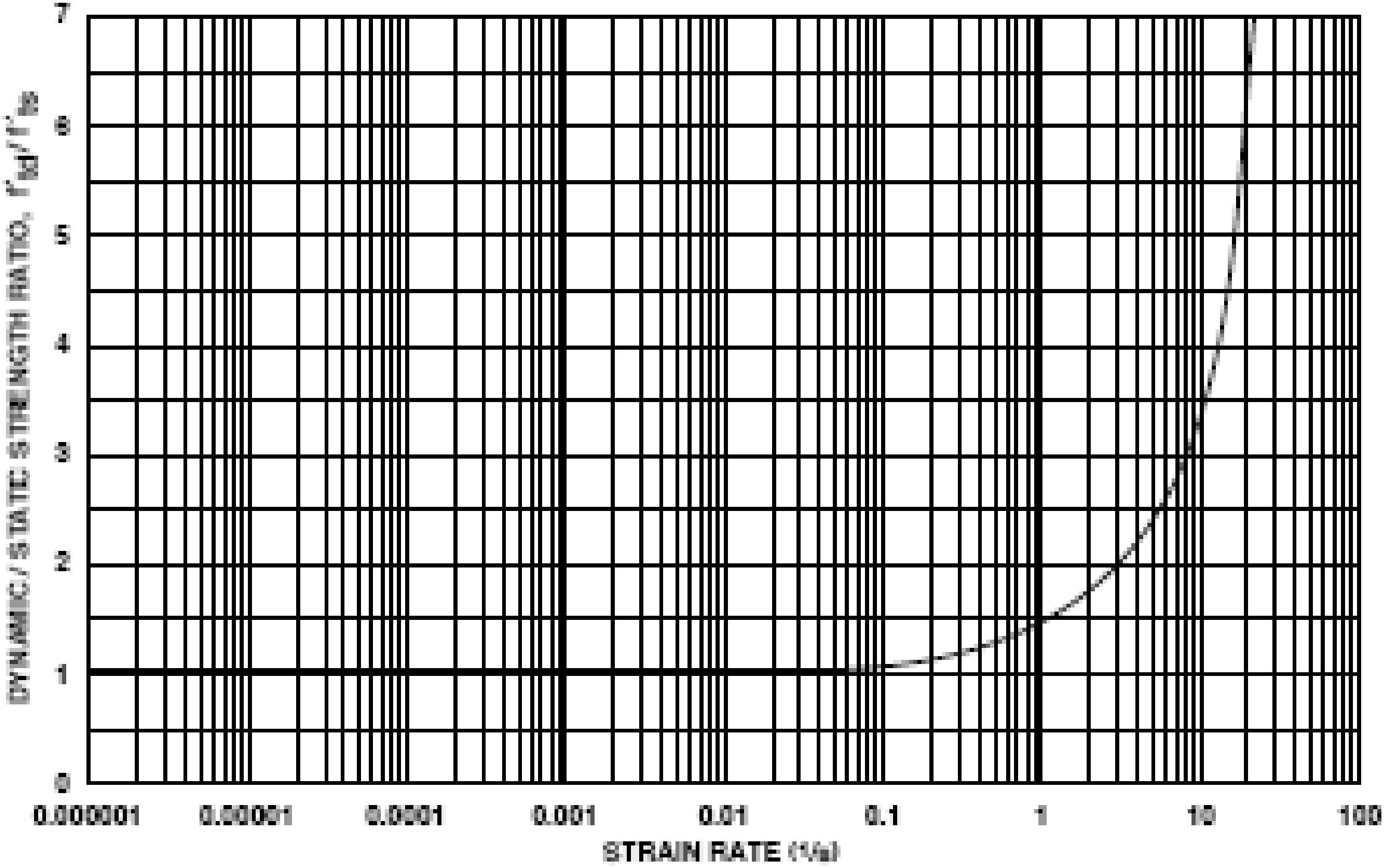

Figure 4-40. Design Curve for DIF for Concrete Splitting Tensile Strength.

Integration

- **Integration Methods**
  - **Constant Velocity**
  - **Newmark Beta with integration constants γ=0.25 and β=0.5**
  - **Both are Implicit Methods**
- **time steps ≥ 0.01 ms - Constant Velocity**
  - **Strengths –fast, good for larger time steps**
  - **Weakness – becomes unstable for small time steps**
- **time steps < 0.01 ms - Newmark Beta**
  - **Strengths – good for small time steps**
  - **Weakness – may become unstable for large time steps**

SDOF Solution Process

- Multiple iterations are performed depending on selection of several analysis enhancements.
  - Material strain rate for automatic dynamic increase factor calculation.
  - Strain hardening
  - Convergence of load mass factor and ductility.
- If convergence conditions were not satisfied a warning message is issued in the text output.

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SDOF Properties

- **Natural Period, $T_n = 2\frac{\pi}{\omega} = 2\pi\sqrt{M/K}$**
- **Mass, M**
- **Stiffness, k**
- **Resistance, R_u**
- **Load-mass Factors, $K_{LM}=K_M/K_L$**
  - **Load factor K_L is the percent of load applied to the analysis**
  - **Mass factor K_M is the percent of Mass applied to the analysis**
- **Equivalent Elastic Deflection, X_E**

Time Step

- Available computer memory is the only limit for number of time steps
- General Rules
  - Time Step based on the lesser of less than 1/10 Natural Period of member and 10 times the smallest time step of the transient load. (Load used in calculation should be checked for clipping of original load)
  - Duration of Calculation based on the greater of the transient load duration or 4 x Natural Period of member.
- SPAn32 allow for user defined time step or a default time step.
- Caveat : Integration routines can be time step sensitive

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Load Definition

- SPAn32 assumes a uniform loading on a member, and relies on other programs for calculation of the transient loading of the structure member
- Airblast
  - Automated Designer
  - Conwep
  - BlastX
- Ground Shock
  - Foil
  - GShock

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Load Definition

1) **External Static Loads are load that is not associated with the mass of the member. It could be associated with live load and some dead loads. It is applied to the structure to set the initial displacement (see separate slide discussion).**

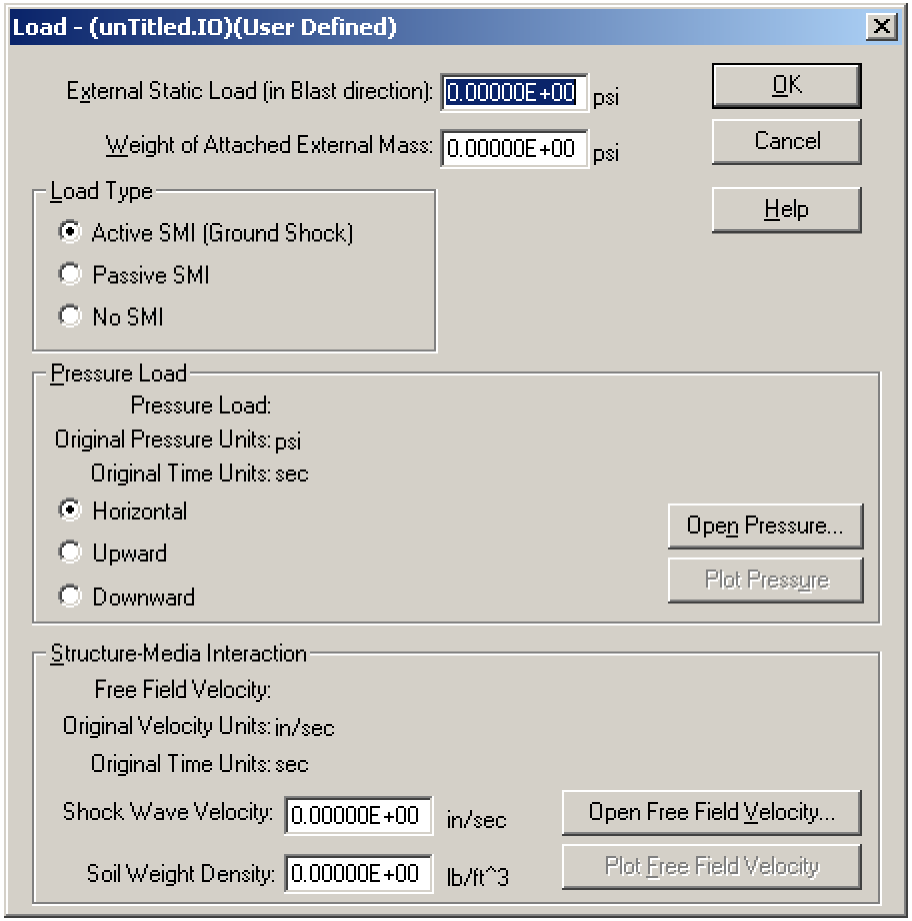
2) **Input the weight of attached mass carried with the structure (see separate slide discussion).**

3) **SMI is soil media interaction and is used for ground shock against a buried structure, conventional construction will typically use No SMI.**

4) **Input direction of pressure load as horizontal, upward or downward.**

5) **Input soil properties for ground shock calculations.**

Additional Loads



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External Static Load and Attached Mass

$$\frac{\left[Resistance\ (x) - \left(P(t) + F_{static} \right) \right]}{\left(K_{LM} \cdot Mass_{member} + K_{LM} \cdot Mass_{attached} \right)} := a(t)$$

- **General differential equation for a SDOF system**
  - **x translation term (acceleration, velocity, displacement)**
  - **$Mass_{member}$ – Mass of the structural member**
  - **$Mass_{attached}$ – This is the mass of attached mass that is coupled to act with the mass of the member.**
  - **K_{LM} – Load Mass Factor**
  - **P(t) – Pressure history**
  - **F_{static} – Static load in blast direction, typically decoupled from the mass of the structural member.**
  - **Resistance(x) – Resistance of the member**

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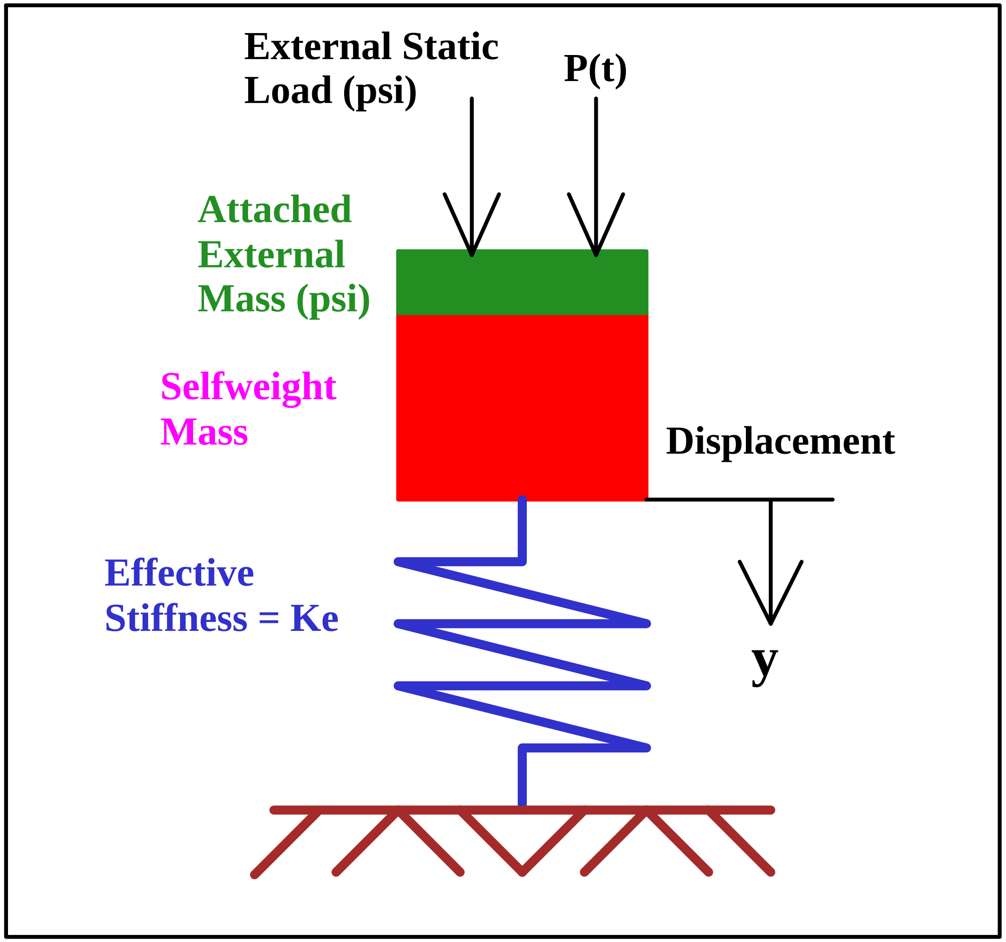
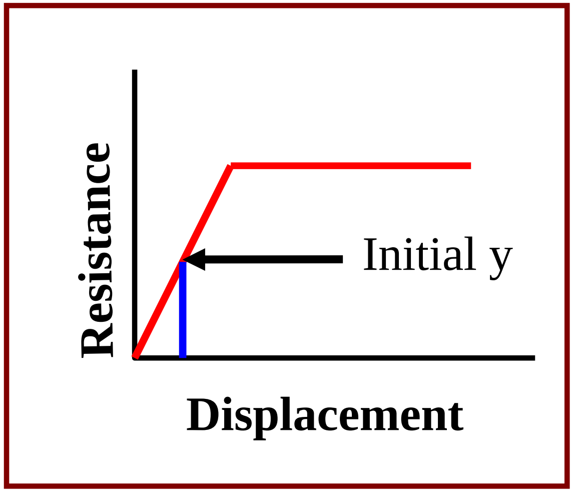
External Static Load and Attached Mass



The external static load is used to calculate the initial displacement of the SDOF model, and then starts the dynamic analysis along the resistance function curve from that point.



Loading Special Cases

- **Pressure Load is applied as a uniform load (force/unit area)**
- **Load Direction is important, especially accounting for the effects of gravity, it also determines the orientation of your member.**
  - **Horizontal**
  - **Upward**
  - **Downward - * static load is applied as a mass, because the load drives it into the member and the combined masses act together. Because of this do not factor in mass effects of the load.**

Graphical Output

- 1. Input pressure load history
- 2. Input soil velocity history
- 3. Interface pressure @ supports
- 4. Interface Pressure @ max deflection point
- 5. Member resistance history
- 6. Member displacement
- 7. Member velocity
- 8. Member acceleration
- 9. Dynamic Reaction History
- 10. Dynamic Resistance vs. displacement

Support Reaction History

$$V_a(t) := C_1 \cdot R(t) + C_2 \cdot P(t)$$

$$V_b(t) := C_3 \cdot R(t) + C_4 \cdot P(t)$$

where,

- **V(t) = Shear history @ supports a and b**
- ***R(t)* = member resistance at time, t**
- ***P(t)* = load at time, t**
- **C_1,C_2,C_3,C_4 are coefficients that determine the proportioning of reactions to account for both loading and inertial forces.**

The support reaction history is used to develop the dynamic load for the supports of the member being analyzed. If you are analyzing a roof member, the dynamic reaction history would be used to design the supporting members for the roof member.

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Comparative Analysis

- **Objective: Verify that the SPAn32 SDOF modeling methodology is comparable to results obtained with DYNA3D**
- **The model was created in DYNA3D using 8 node cube shaped brick elements with 4 elements through the slab thickness. The DYNA3D model was developed to match the SPAn32 resistance function as closely as possible. Matching the resistance function provided a control in the analysis and isolates the SPAn32 SDOF methodology for verification.**

Resistance Function

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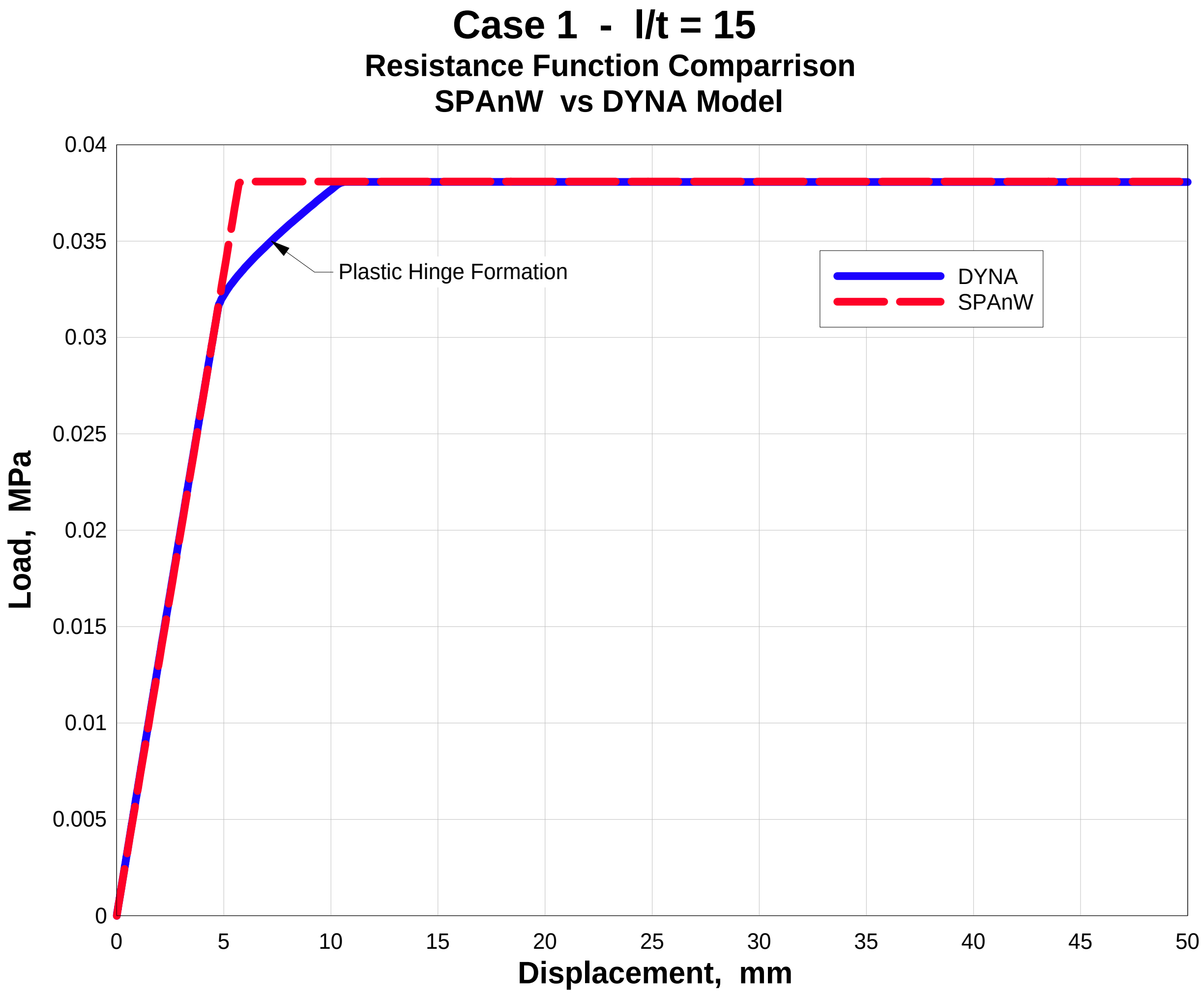

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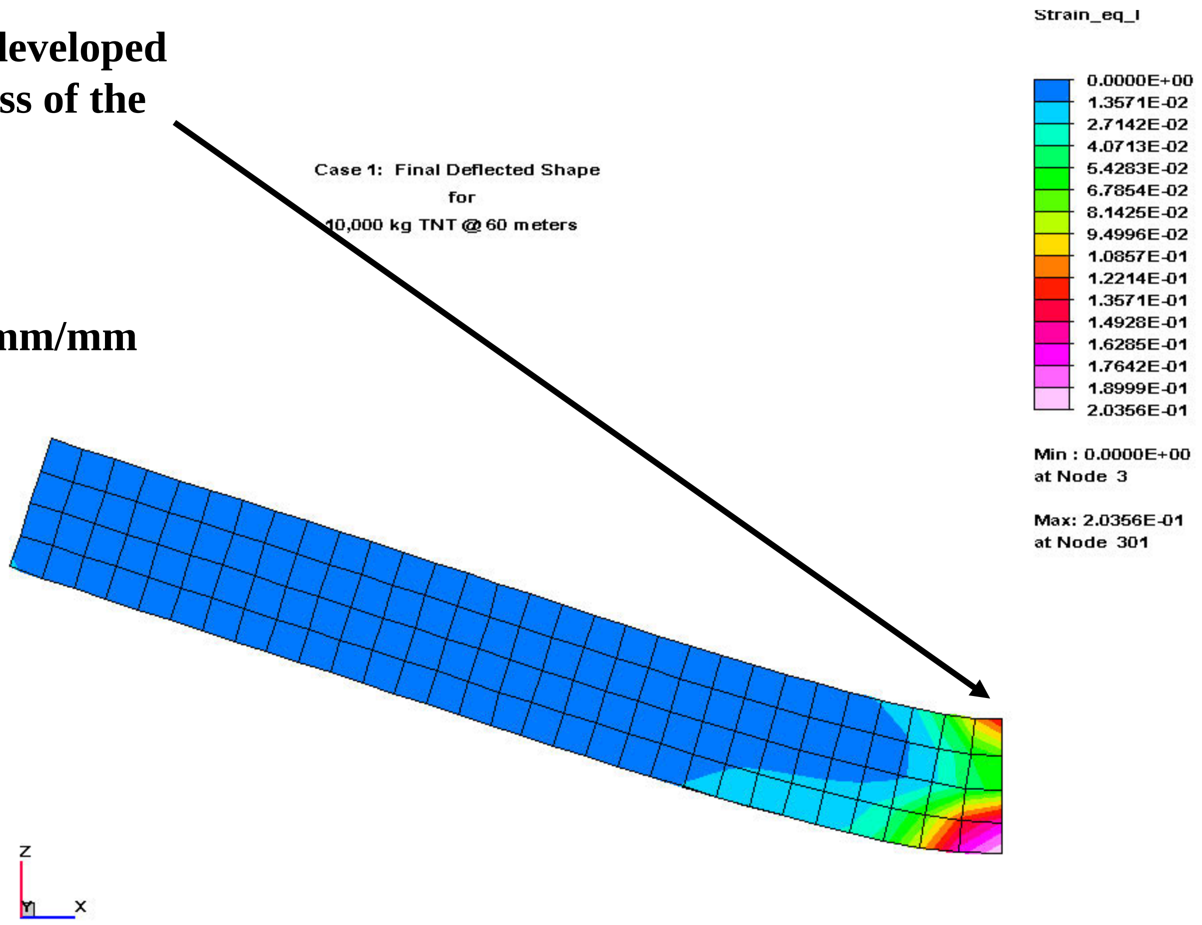
DYNA3D Model

- Plastic Hinge fully developed through the thickness of the member
- f_y = 4.272 MPa
- E=17550 MPa
- $\varepsilon_y = f_y/E$=2.434e-4 mm/mm



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Peak Response Comparison

Load, kg @ m	SPAn32, mm Peak response	SPAn32 Ductility	DYNA3D, mm Peak response	SPAn32/ DYNA3D ratio
100@60	2.86	0.49	2.85	1.004
200@60	4.36	0.75	4.33	1.007
500@60	8.10	1.40	8.66	0.935
1000@60	17.73	3.06	19.16	0.925
2000@60	50.09	8.63	52.93	0.946
5000@60	217.6	37.5	221.5	0.982
10000@60	653.7	112.7	652.1	1.002
SPAn32 time step = 0.1 ms l/t = 15:1, thickness 300 mm, unsupported length = 4500mm, ρ = 0.3 %, elastic limit = 5.8 mm				

Results Summary: SPAn32 typically under-predicted the response calculated from DYNA3D. This can be partly attributed to the differences in the resistance functions. Closest agreement occurred when the ductility was less than one. It is also noted that beyond the elastic region agreement in peak response improves as the ductility increases. This trend may be occurring, as differences in the resistance function become less of a contribution to the peak response.

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Future Enhancements to SPAn32

- **New Enhancements**
  - **generation of Pressure-Impulse curves for given response.**
- **Enhancements Partially Completed (need funding)**
  - **12 steel model for reinforced concrete to provide more accurate support conditions.**
  - **Openings in a slabs and the resulting yield lines.**
  - **A user defined multi-step resistance function 3)**
  - **Runge-Kutta Explict Integration.**
- **Further testing and validation of the SPAn32 program are needed, specifically validation with test data.**

SPAn32 Availability

- **Distribution restricted**
  - **Distribution Statement C**, Distribution authorized to U.S. Government agencies and their contractors: Critical Technology.
  - **WARNING:** This document contains technical data whose export is restricted by the Arms Export Control Act (Title 22, U.S.C., Section 2751, et seq) or the Export Administration Act of 1979, as amended, Title 50, U.S.C., App. 2401 et seq. Violators of these export laws are subject to severe criminal penalties. Disseminate in accordance with provisions of DoD Directive 5230.25.

- **U.S. Government agencies and their contractors.** also(UK, Australia, Canada, New Zealand, limited NATO)

How to Get SPAn32?

- **Public Site:**
  - **https://pdmcx.pecp1.nwo.usace.army.mil/software/span32/index.php**
  - **Instructions how to get SPAn32**
  - **Password instructions**
  - **Registration Key**
- **FOUO Site: (password protected)**
  - **https://forge1.pecp1.nwo.usace.army.mil**
  - **SPAn32 support**
    - **Bug tracking**
    - **Help**
    - **Frequently asked questions**
    - **Documentation**
    - **Download SPAn32 and supporting files**

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Questions ?